No. 17

McGILL UNIVERSITY

PAPERS FROM THE DEPARTMENT
OF
GEOLOGY

No. 22.—On the Need of a Topographical Survey of the Dominion of Canada

BY

FRANK D. ADAMS

No. ~~23~~ 17.—On a New Nepheline Rock from the Province of Ontario, Canada

BY

FRANK D. ADAMS

No. ~~24~~ 23.—An Investigation into the Elastic Constants of Rocks

BY

FRANK D. ADAMS

AND

ERNEST G. COKER

Reprinted :
No. 22, from Jour. Can. Min. Inst., part of Vol. IX, with paging.
17. No. ~~23~~, from Amer. Jour. Sci., 4th Ser., Vol. XVII, No. 100, April, 1904, pp. 269-276.
23. No. ~~24~~, from Amer. Jour. Sci., 4th Ser., Vol. XXII, No. 128, August, 1906, pp. 95-123.]

Montreal, 1907

Art

[P
Surv

In
the
ship
of K
renc
man
one
Onta
—wh
logic
sever
neph
of m
of th
line-b
geolo
write
expec
for c
Prof.
visite
the m
Burea
Wh
the C
Pirsso
their
descri
alkali
very p
allied
the pa
trict r
the a
this o
any o
compo
rare, h
taking
of the

---

* F. I
in the T

Art. XXIV.—*On a New Nepheline Rock from the Province of Ontario, Canada*; by Frank D. Adams.

[Published with the permission of the Acting Director of the Geological Survey of Canada.]

In a paper which appeared in this Journal some years since,* the discovery of a large body of nepheline syenite in the township of Dungannon, in eastern Ontario about 85 miles northwest of Kingston, was recorded. As stated at the time, this occurrence is of large dimensions and the rock constituting it is in many respects remarkable in character. The discovery was one of the first results of a geological survey of this part of Ontario—at that time, geologically speaking, a *terra incognita*—which had just been undertaken by the writer for the Geological Survey of Canada. As the survey was continued during several succeeding summers, many additional occurrences of nepheline syenite were discovered and mapped, and as the result of more extended study the area is now known to present one of the most extensive and interesting developments of nepheline-bearing rocks which are known to occur anywhere. The geological maps of the area in question, by Dr. Barlow and the writer, are now completed and are being engraved, and it is expected that they, with the accompanying report, will be ready for distribution shortly. During the progress of the survey, Prof. Miller and Dr. Coleman of the Ontario Bureau of Mines visited the district and described certain of the occurrences of the nepheline syenite in papers published in the Reports of the Bureau and elsewhere.

When engaged in elaborating their Quantitative System for the Classification of Igneous Rocks, Messrs. Iddings, Cross, Pirsson, and Washington found that certain subdivisions of their scheme had no representatives amongst the rocks hitherto described. One of these subdivisions was Order 8 of the Peralkalic Persalanes, to which would belong nepheline syenites very poor in feldspar and very rich in nepheline, with perhaps allied varieties of related rocks. As it has been mentioned in the paper on the Dungannon nepheline syenite, that in that district rocks composed almost exclusively of nepheline occurred, the authors of the Quantitative Classification suggested that this order might be called Ontarare. No analysis, however, of any of these rocks had then been made, so that their precise composition remained somewhat doubtful. The name Ontarare, however, was given to the order, the present writer undertaking to make good the claim of the Province to the bestowal of the name, by describing an Ontarare from the district in question which would serve as a type for the order. It is the purpose of the present paper to make good this undertaking by describing the first Ontarare.

---

* F. D. Adams: On the Occurrence of a Large Area of Nepheline Syenite in the Township of Dungannon, Ontario, this Journal, July, 1894.

The nepheline syenites of eastern Ontario, while always presenting the same general character, are represented by many varieties. The rock is usually coarse in texture, while in some of its pegmatitic developments the most extraordinary size of grain is attained. In one occurrence near the village of Gooderham in the township of Glamorgan, consisting of nepheline, albite, and an occasional individual of lepidomelane, the nepheline masses—for the most part single individuals—are often as much as three feet in diameter; while in several other occurrences nepheline syenite pegmatites of almost equal coarseness of grain have been found. The rock, furthermore, differs from that of most nepheline syenite occurrences, in that it commonly presents a more or less distinct foliation or gneissic structure, which foliation is not due to crushing *in situ* with the development of cataclastic structure, but is produced by a parallel arrangement of the constituent minerals, which arrangement seems to be a primary one or is at least unaccompanied by pressure phenomena.

The nepheline syenites of the region, furthermore, show a wide variation in mineralogical composition, several varieties often occurring together in the same mass, forming rude bands which coincide in direction with the foliation, thus serving to accentuate this and render it more pronounced.

The iron-magnesia constituent which is most commonly present is hornblende, represented by alkali-rich varieties, of which hastingsite may be taken as a type.* Pyroxene, however, replaces this in some cases and biotite in others. In some places these minerals preponderate over the colorless constituents of the rock and in these varieties garnet locally occurs in considerable amount. These dark bands, or schlieren, are, however, not very common and the prevailing facies of the rock is one which is light in color owing to the abundance of feldspar and nepheline. In certain occurrences other minerals which usually play the part of accessory constituents—such as corundum, sodalite, and cancrinite—attain more prominence, the first mentioned mineral now being very extensively mined in an occurrence of the syenite, near Combermere; while some of the largest masses of sodalite† which have ever been found are those which have been obtained from the nepheline syenite in the township of Dungannon, to the east of the village of Bancroft.

---

* F. D. Adams and B. J. Harrington: On a New Alkali Hornblende and a Titaniferous Andradite from the Nepheline Syenite of Dungannon, Ontario, this Journal, March, 1896.

† B. J. Harrington: On Nepheline, Sodalite and Orthoclase from the Nepheline Syenites of Dungannon, Hastings Co., Ontario, this Journal, July, 1894.

These nepheline syenites occur cutting the rocks of the Grenville series, which consist largely in this district of crystalline limestones. Their wall rock consequently in almost every case is limestone. The only occurrence which is not directly associated with large masses of this rock is a large isolated intrusion in the township of Methuen, where the country rock is granite and amphibolite, the latter holding only a few small limestone bands.

With this is connected one of the most curious phenomena presented by these nepheline syenites, namely, the presence in them almost everywhere of calcite. This calcite when appearing in the analysis of a rock at once suggests an advanced stage of alteration, since calcite when found in igneous rocks is almost invariably a secondary product. In other cases calcite in plutonic rocks has been supposed to occupy miarolitic druses and to have been deposited in these by percolating waters.

A very careful examination, however, of the calcite-bearing occurrences of nepheline syenite in the various parts of this area has clearly shown that in the case of these rocks the calcite represents inclusions of the crystalline limestone penetrated by the intrusion, a fact which receives additional substantiation in the fact that in the Methuen occurrence, when the wall rock is not limestone, the nepheline syenite does not contain calcite. Along the borders of the intrusion, the nepheline syenite is seen to eat into the limestone and to enclose large masses, the constituent minerals of the syenite growing into the substance of the limestone, often with well defined crystal terminations. These masses often show a coarsening in grain as a result of the metamorphic action of the intruding rock. On receding from the contact, the inclusions become less numerous and smaller, and eventually the large masses are disintegrated into separate individuals of calcite or small groups of calcite grains. These, under the microscope, can be seen as rounded, often perfectly round, inclusions completely enclosed in a single individual of nepheline or other constituent of the nepheline syenite, or lying between other constituents of the rock, which latter can be seen to have grown into the calcite. In these cases all the minerals are perfectly fresh and unaltered, and, while the constituent minerals of the nepheline syenite rarely show pressure phenomena, the calcite individuals are often seen to be much bent and twisted and to display marked strain shadows, the movements displayed being those which overtook the limestone before the intrusion of the syenite into it.

It being, therefore, clearly recognized that the calcite is something foreign to the original magma and that it merely exists in the rock in the form of inclusions, in calculating out the mineral composition of the rock the calcite is set aside and the primary magma regarded as having the composition of the calcite-free rock. The nepheline syenite of the island of Alnö,* it may be mentioned, is characterized by the presence of calcite which is not of secondary origin, whose mode of occurrence in many respects is very similar to that of the calcite found in the nepheline syenite of Ontario.

Seven typical varieties of the rock have been analyzed and these serve to show the range in composition displayed by the magma. Of these, three were found to belong to the class of the Persalanes and to the sub-class Pesalone; using the nomenclature recently proposed by Messrs. Cross, Iddings, Pirsson and Washington,† while three others belong to the class Dosalane and the sub-class Dosalone. One is a Phlegrose, one a Vulturose and one a Miaskose, while two are referable to Essexose and one is a Kallerudose. These will be fully described in the forthcoming Report to which reference has already been made.

In addition to these there is the rock described in the present paper. This is a variety of the nepheline syenite which is almost free from feldspar and which consists essentially of nepheline and the iron-magnesia constituent, in this case hornblende. It occurs in the township of Monmouth, about 25 miles west of the township of Dungannon. Here, on lots 9, 10, 11 and 12 of ranges VII to VIII, a mass of nepheline syenite breaks up through a great band of crystalline limestone. The southern limit of this mass is unfortunately mantled by drift, so that its extension in this direction is somewhat uncertain. It has, however, the form of a flattened ellipse, the longer diameter measuring one mile and the shorter diameter about half a mile, and is completely surrounded by the limestone. The mass holds many inclusions of the limestone through whose shattered mass it penetrates. These included limestones frequently are coarsely crystalline and are more or less impure from the presence of secondary silicates developed by the contact action. These masses have the appearance of being in process of replacement by the intruded magma. The nepheline syenite in some places along its contact with the limestone is rich in hornblende, but elsewhere along the border it contains but a very small proportion of the dark constituent, so that no distinct endomorphic action can be traced to the influence of limestone. In some few places, however, near the limestone the syenite holds scapolite. Taking the intrusion as a whole it may be said to consist essentially of albite, nepheline and hornblende, but as is so frequently the case in this region it shows a wide variation in relative proportion of the constituents in different places. In some places it is rich in feldspar, while elsewhere the nepheline almost entirely replaces this mineral. In the latter case the nepheline is usually associated with a considerable amount of hornblende, in addition to which in many cases a small amount of red garnet is present.

---

* Hogbom, A. G.: Ueber das Nephelinsyenitgebiet auf der Insel Alno. Geol. Fören. i. Stockholm Förh., Häft. 2, 1895, p. 140.

† Quantitative Classification of Igneous Rocks, University of Chicago Press, Chicago, 1903.

The feldspathic and feldspar-free varieties run in rudely parallel bands or schlieren. These are often several feet in width and may be traced for several hundred yards along the strike of the banding. They represent distinct magmas resulting from extreme differentiation. The rock here described was collected from one of the bands six feet in width and several hundred yards long.

The rock is coarse in grain and consists essentially of white nepheline and black hornblende, the former preponderating largely. It thus has a rather striking appearance on the fresh fracture. On the weathered surface the contrast presented by the two minerals is less striking, as the nepheline assumes a pale gray color. Under the influence of the weather, the nepheline, as is always the case with these rocks in this district, presents the appearance of having been dissolved away, the weathered surface being smooth and recessed, the hornblende and the accessory feldspar and cancrinite of the rock standing out from the surface of the nepheline.

Under the microscope the rock is seen to consist essentially of nepheline and hornblende, with plagioclase, cancrinite, and calcite as accessory constituents, as well as sodalite, apatite, sphene, biotite, pyrite and iron ores, these latter minerals being present in extremely small amounts.

The Nepheline occurs in large well-defined grains, presenting the usual characters displayed by the species. It is clear and fresh.

The Hornblende is green in color, the pleochroism and absorption being as follows: $\mathfrak{a}=$ pale greenish yellow. $\mathfrak{b}$ and $\mathfrak{c}=$ very deep green. The absorption is $\mathfrak{c}=\mathfrak{b}>\mathfrak{a}$. The maximum extinction observed in the sections of the rock was 19°. It is an alkali hornblende, containing less iron than hastingsite, but, like it, as shown by the calculation of the analysis of the rock, belonging to the division of the Syntagmatites.

The Plagioclase is present only in very small amount and is in some cases untwinned, while in other cases it shows a faint, polysynthetic twinning. In thin sections it bears a very close resemblance to the nepheline, and when untwinned it is difficult in all cases to distinguish the two minerals. When a section is treated with acid and etched, however, the plagioclase is seen to occur in individuals of a more or less rounded form or with curving outlines, lying between the nepheline grains or enclosed in the latter. The feldspar isolated from another variety of the rock in the same occurrence was found to be albite, and this feldspar has, therefore, been taken as albite in calculating the Mode of the rock.

The amount of Cancrinite present varies very considerably in different specimens of the rock. In the specimen analyzed about 5 per cent was found. In other specimens more is found, although in no case is it very abundant. It is clear and colorless, but is at once distinguished from the nepheline when examined between crossed nicols by its much higher polarization colors, which in thin sections frequently rise to a blue of the second order. It is clear and free from interpositions and in convergent light is seen to be uniaxial and negative. It also shows a slight but distinct dispersion of the bisectrices, giving a brownish and a bluish tint on either side of the position of maximum extinction. When separated by Thoulet's solution, the mineral was found to have a specific gravity between 2·48 and 2·44, and to be readily decomposed when heated with dilute hydrochloric acid with the evolution of carbonic dioxide and with subsequent gelatinization. The cancrinite occurs in the nepheline in the form of narrow strings or more rarely in little bunches of grains. These usually follow the course of minute cracks or cleavage lines, but also are frequently seen to follow the boundaries of individual grains of nepheline on their contact with grains of other minerals. Thus between crossed nicols they appear as a brilliant edging about hornblende individuals or about calcite inclusions in the nepheline, the small prismatic individuals of cancrinite being arranged with their longer axes at right angles to the contact or to the course of the crack, as the case may be. The cancrinite has the appearance of being an alteration product of the nepheline.

The Calcite occurs in large single individuals, which are found as inclusions in both the hornblende and the nepheline. The single individuals are often perfectly circular in outline, and the enclosing mineral is perfectly fresh and unaltered and is sharply defined against them. In other cases the same large calcite individuals lie between the other constituents of the rock, in all cases having the character of inclusions. They generally show very marked strain shadows, while the other constituents show but little or no evidence of pressure phenomena.

The Apatite is found as occasional more or less rounded individuals, enclosed in the nepheline or hornblende, but, like the other accessory constituents, merits no especial description.

An analysis of the rock made for me by Mr. M. F. Connor gave the following results:

SiO_2	39·74
TiO_2	·13
Al_2O_3	30·59
Fe_2O_3	·44
FeO	2·19
MnO	·03
CaO	5·75
MgO	·60
K_2O	3·88
Na_2O	13·25
CO_2	2·17
SO_3	trace
Cl	·02
S	·07
H_2O	1·00
	99·86

If, following the methods of the Quantitative Classification, the Norm of the rock be calculated, that is to say the proportion of standard minerals which would give a magma of this composition, or in the form of which the rock under other conditions of cooling might have solidified, this is found to be as follows:

Anorthite	12·51
Nepheline	67·72
Leucite	8·28
Olivine	3·70
Akermanite	·40
Magnetite	·70
Ilmenite	·30
Pyrite	·14
Calcite	4·92
	98·67
Water	1·00
	99·67

This gives the rock the following position in the Quantitative Classification:

Class 1—Persalane.
Order 8—Ontarare.
Rang 2—(Domalkalic).
Sub-rang 4—(Dosodic).

As this is the first Ontarare which has been described, the prorangs and sub-rangs have received no names as yet. It is proposed, therefore, to call rang 2, Monmouthase, and sub-rang 4, Monmouthose, from the township of Monmouth in which this rock is found, while, as an ordinary designation, the name Monmouthite may be applied.

The Mode, or actual mineralogical composition of the rock, is quite different from the Norm, as given above, no leucite, anorthite, olivine, or akermanite being actually present. The mode is *abnormative** to a striking degree.

The Mode is as follows:

Albite	1·83
Nepheline	72·20
Sodalite	·28
Cancrinite	5·14
Hornblende	15·09
Hematite	·50
Calcite	3·12
Pyrite	·14
	98·30
Water	·50
Excess of Al_2O_3	1·20
	100·00

In calculating this mode the nepheline is taken as consisting of soda nepheline and kaliophyllite, in the proportions of 5 to 1, which is the composition of the nepheline of the nepheline syenite occurring further to the west in the area of the township of Dungannon.† One-half of the water found in the analysis is considered as being present in the cancrinite, the remainder being regarded as belonging in part to the hornblende and existing in part as hygroscopic water. This gives cancrinite in about the proportion in which it seems to be present in the thin sections of the specimens analyzed.

The various bases not required by the other minerals and remaining over to form the hornblende, are present in the proportions required to form syntagmatite; which are the proportions in which these bases are found in the hastingsite of the Dungannon nepheline syenite. The hornblende has accordingly been calculated as syntagmatite, using the theoretical values given by Zirkel: Lehrbuch der Petrographie, vol. i, p. 303. This accounts for the existing percentages of all the constituents of the rock, with the exception of an excess of 1·20 per cent of alumina.

Of the rocks hitherto described, those which bear the closest resemblance to Monmouthite are the Urtites of the Peninsula of Kola.‡ These, however, belong to the class of the Dosalanes.

Geological Department, McGill University, Montreal, P. Q.

---

* See Quantitative Classification (loc. cit.), p. 150.

† B. J. Harrington: loc. cit.

‡ W. Ramsay: Das Nephelinsyenitgebiet auf der Halbinsel Kola, Fennia. 15, No. 2. p. 22.

Subject to Revision. *Advance copy sent for discussion.*

Journal Canadian Mining Institute. Part of Vol. IX.

ON THE NEED OF A TOPOGRAPHICAL SURVEY OF THE DOMINION OF CANADA, PARTICULARLY WITH REFERENCE TO THE DEVELOPMENT OF THE ECONOMIC RESOURCES OF THE DOMINION.

By FRANK D. ADAMS, McGill University, Montreal.

At no time in its past history has Canada shown such a rapid growth and development as during the last half dozen years. Settlers are pouring in and new districts are being opened up in all directions, and yet the Dominion is without that great national asset which is a prime necessity for the satisfactory and economical development of our natural resources—a topographical map of our country. Canada is in this respect outstripped by every civilized country in the world. We are, in fact, like a householder who is engaged in furnishing his house and yet has no adequate knowledge of the number and dimensions of its rooms, and only a most hazy idea of the size and character of the grounds surrounding it.

The proof that the people of every other civilized country regard such a survey as necessary, is shown by the fact that they have long since, in every case, established a regular ordinance or other government survey, which has prepared and published topographical maps of these respective countries, and if we consider for a moment the uses of such a map and the great waste of public and private moneys which would be avoided by its preparation, it will at once be seen that the construction of a good topographical map of Canada is one of the most pressing needs of the Dominion at the present time and one which will make itself felt with ever increasing insistence as the country opens up during the next few years. The preparation of such a topographical map of the Dominion would be a public work of enormously greater value than many other pieces of work which are now being undertaken by the government and would cost much less. It would at the same time be an undertaking which would provide a permanent asset for the country and one which would pay a high rate of interest on the original expenditure, in the services which it would render in a thousand ways both to our Government and to the citizens of the Dominion. The singular fact is that Canada is surveying her country incessantly, and yet has no good map of it. Whenever a new project comes up, the land is re-surveyed. If a railway is planned, almost the whole work of examination of the country must be done from the start. If an engineer wishes to ascertain with precision how much electric energy can be derived from the waters of any river, it is necessary for him to go and make a map of the river basin as well as to gauge the stream. In 1903 one portion of the Grand Trunk Pacific controversy would have been avoided if there had existed a proper map of northern New Brunswick. Expert opinion at that time tended to the belief that the shape of the hills in that part of the Dominion would render it very difficult to obtain low grades across them. A map such as the United States map of the Adirondacks would have settled the question at once and have shown that the danger was imaginary.

Furthermore, we have a dozen or more different branches of the government's service carrying on isolated and independent surveys, often covering the same areas, all more or less imperfect and by their duplication entailing a great loss of time and money. Thus, in the North West Territories, within a comparatively limited district, surveys have been made by the following branches: Topographical Surveys and Timber & Mines branches of the Department of the Interior; Geological Survey; Department of Railways; Indian Department; &c. In the older provinces this state of affairs is further complicated by the existence of surveys under Provincial Government auspices, surveys by the Public Works Department and by railway companies, charts of the coast by the Department of Marine and the Admiralty, and other minor surveys which it is not necessary to enumerate.

The remedy for this state of affairs is admirably outlined in the recommendation of the Civil Service Commission of 1882 as follows:—

"Your Commissioners find that maps for various purposes are prepared in several Departments and, by the evidence produced, it is shown that differences frequently occur in maps of the same district when issued by more than one Department. To prevent this and to promote accuracy and security, it is recommended that a special cartographic branch be created and that the duty of this branch of the public service should be to issue reliable maps of the various parts of the Dominion."

Now, what is the nature of a good topographical map such as that which we need for the proper development of our varied resources, and what will it show? It will show accurately, in the first place, the position of every natural feature in the district surveyed. In the second place, it will show with equal accuracy the relief (height, slope, &c.) of every part of the area in question, and, thirdly, it will also show the varied results of the human occupation of the country—towns, villages, roads, &c., &c. The precise character of such a map will be more clearly seen if we consider the case of the excellent series of topographical maps which are now being issued by the Geological Survey of that country whose conditions most nearly approach to those of our own, namely, the United States.

The features exhibited on the topographical maps of the United States Geological Survey are:*

1. Hydrography, or water features, as ponds, streams, lakes, swamps, &c., which are printed in blue.

2. Hypsography, or relief of surface, as hills, valleys, and plains, which are printed in brown.

3. Culture, or features constructed by man, as cities, roads, villages, and the names and boundaries, which are printed in black.

This combination of colors renders these topographic maps readily legible. On the reverse of each sheet is a description of the mode of reading the map, and a legend, or series of conventional signs, indicating how the various facts shown on the map are represented. All these conventions are self-explanatory and are readily understood and interpreted by the layman, except, perhaps, the brown "contour" lines.

---

* See "The United States Geological Survey"—Bulletin No. 227 U.S. Geol. Survey, Washington, 1904, p. 57. The author also desires to acknowledge his indebtedness to James White, C.E., Dominion Geographer, for information employed in preparing the present paper.

These contours are lines of equal elevation—lines along which the ground would be touched by the border of a water surface (of the ocean, for instance) if it were repeatedly raised by a given amount. Contour lines express three features of relief (1) elevation, (2) horizontal form, and (3) grade or slope.

The topographic maps of the United States Geological Survey represent, beside the natural features mentioned, such artificial features as are of a public or more enduring nature, as state, county, township, and city boundary lines; reservation, land-grant, and public-land township lines; railways, streets, roads and paths; bridges, ferries, dams, locks and wharves; the location of permanent bench marks and triangulation stations, and the positions of light-ships and light-houses, mines, shafts and tunnels. The only private features shown are houses, factories, stores, etc., no barns or outbuildings being represented.

As the outlines of wooded areas are to be indicated on these maps, National and State foresters find them invaluable as a base for classifying the woodlands and recording the nature and quantity of the various trees and the relations of the wooded areas to highways of transportation, as railways, streams, &c. These maps are, of course, essential to detailed geological studies and to investigations concerning mineral resources, water power, and land reclamation.

A good topographic map renders unnecessary a special survey for each new need. Prior to the existence of such maps every city was obliged to expend large sums in water-supply surveys. At far less cost the topographic map shows not only all these important local features, but also the relations between the artificial features in the immediate neighborhood and the topography and culture of the surrounding country, and thus broadens the scope of every such investigation.

It is noteworthy, that with the exception of one map of a portion of Manitoba and a few of portions of British Columbia and the Yukon District, none of the maps hitherto issued by the Canadian Government show that most important feature of a topographical map, the relief of the surface.

The advantages of an accurate topographical map of our country might be set forth as follows:

(1) Educational—(a) By promoting an exact knowledge of the country; (b) by serving teachers and pupils in geographical studies.

(2) Practical—As preliminary maps for planning engineering projects. Highways, electric roads, railroads, aqueducts and sewerage plants may be laid out on them, and the cost of preliminary surveys may be saved. Areas of catchment for water supply, sites for reservoirs, and routes of canals may be ascertained from these maps.

(3) Political—In all questions relating to political or legislative matters. For these purposes they afford accurate information as to the relation of boundaries and towns to natural features.

(4) Administrative and Military—In all questions relating to Federal or State administration of public works, as canals, reservations, parks, highways, postal service, and as military base maps on which to plan works of offence, defence, camps, marches, &c.

(5) Statistical—As base maps for the graphic representation of all facts relating to population, industries, products or other statistical information.

(6) Economic—As a means for showing the location, extent, and accessibility of lands, waters, forests, and valuable minerals. In this respect these maps are indispensable to State and Federal bureaus, and to owners, investors and corporations.

A specific case of the saving effected by the use of such a topographical map is given by President C. R. Van Hise:—

"A railroad company owned two lines in northern Michigan and Wisconsin which they wished to connect by a branch some forty miles in length. The country was unknown between the two lines. Under my direction topographical, geological and timber maps were made of about twelve hundred square miles, at a cost of about $25,000. After these maps were made the line was laid down in the office with a beautiful grade, and with the best location possible with reference to iron ore and timber. One of the features of the line was that it crossed a bold trap range which was supposed to be impossible to cross except at great expense, by finding gaps through it. This line crossed the branch of another railroad about twelve miles long, which because it went around the end of this trap range, when the gap was directly in front of it, was at least two miles longer than necessary. These additional two miles of road cost fully as much as the entire survey of the twelve hundred square miles.

It is my profound conviction, if topographical surveys were made of broad zones where it is expected to project railroads through rough forested country, that many times the cost of the survey would be saved in the construction of the railroads, besides getting a better location both as to resources and to grade than is possible by the random methods employed at the present time for the locating of railroads in rough forested country."

The construction of a topographical map of the United States has been assigned to, and is being admirably carried out by, the Geological Survey of that country; the Coast and Geodetic Survey confining their operations to the mapping of the coastline and the strip of country bordering it.

The general plan adopted for mapping the United States consists in dividing the country into quadrilateral areas (called "quadrangles") bounded by parallels and meridians of latitude and longitude* Three regular scales and a series of special scales have been adopted. For reconnaissance maps in the rougher portions of the Far West and Alaska the scale of 1:250000, or nearly 4 miles to the inch, is standard. For the general map of the United States the scale of 125,000, or nearly 2 miles to the inch, is standard. In the more densely populated districts—the Atlantic coast, parts of the Central region, and portions of the Pacific coast—the scale of 1:62500, or about 1 mile to the inch, is standard. These scales are readily convertible into the metric and other systems understood in foreign countries. However, at the bottom of each sheet the scale is expressed in three ways—by a graduated line representing miles and parts of miles in English inches; by a similar line indicating distance in the metric system, and by a fraction. Special larger scales, from 2,000 feet to the inch up to 100 feet to the inch, are used for detailed surveys of important mining regions, for the planning and construction of irrigation projects, etc.

---

* See Bulletin 227, U.S. Geol. Survey, already cited, pp. 60–63.

The maps are printed on sheets approximately 13 by 17 inches. The area represented on a sheet on the 1 mile scale is 15 minutes of latitude and of longitude, or, on the average, about 225 square miles; on the 2-mile scale, 30 minutes of latitude and of longitude, or about 1,000 square miles; and on the reconnaissance or 4-mile scale, 1 degree of latitude and of longitude, or approximately 4,000 square miles. An atlas sheet is known by the name of the most important town, city or natural feature within the quadrangle represented.

The methods of field work followed in making these topographic surveys are as follows: At rather distant intervals through the territory under survey, say 10 to 25 miles, monuments are established, the positions of which are accurately determined by trigonometric methods and recorded in degrees of latitude and longitude. For this purpose base lines from 4 to 6 miles in length are measured with great accuracy, the location of one end of each base and of its azimuth being determined astronomically. From this base a net work of triangulation is expanded to hilltops, mountain summits and other prominent objects, the positions of which are carefully determined by computation. This is the special work of the section of triangulation and computing. All existing triangulation by other organizations is utilized. Where the nature of the country renders the extension of triangulation impracticable, primary control is extended by means of careful lines of traverse.

The resultant positions are plotted accurately, by latitude and longitude, on large sheets of drawing paper, approximately 18 by 24 inches in size, called plane-table sheets. To each topographer is assigned, as his season's work, the mapping of one or more quadrangles, and he enters the field equipped with the plane-table sheet, which has on it only the trigonometric positions. A field party usually consists, in addition to the topographer in charge, of one skilled assistant, of the grade of assistant topographer or topographic aid, and several surveyors temporarily employed as traversemen or levelmen, with the necessary rodmen, teamsters, cooks, and other camp hands where subsistence is by camping.

The levelmen proceed to run lines of spirit levels over the various roads and paths, to determine elevations. The higher order of primary leveling, elsewhere described, is run at such distances apart that at least two permanent bench marks may be established in each township or equivalent area 6 miles square. Levels of a less degree of accuracy are then run over every road, and where these are too far apart, say a mile or more, the levels are run over trails or across country, either by spirit level or by vertical angulation with stadia.

Meantime the topographer is extending a graphic triangulation on his plane-table board, the object of which is to locate precisely, on his sheet, points all over the area under survey. These include signals placed upon prominent hills, church spires, lone trees, and other definite objects; or in case the country is of such nature that it cannot be controlled by triangulation, he has careful lines of traverse, by stadia or steel tape, run at intervals of about 5 miles and connected with the lines of primary traverse previously run across the sheet. At the same time, the traversemen, equipped with lighter plane-table boards on which are smaller sheets representing about one-fourth of the whole quadrangle, are running graphic traverse lines over all roads and paths and across country, with a view to locating the cultural features, the courses of streams, the outlines of lakes, etc. On highways the distances are measured by counting the revolutions of a wheel, the circumference of which is known, or by stadia, and in dense woods by tape.

On the completion of this primary control and coincident with its preparation the topographer transfers to a clean sketch sheet the primary control points and adjusts to these the positions determined by secondary plane-table triangulation or stadia or tape traverse. The less accurate traverse lines are then adjusted between these more accurate positions and the elevations procured by spirit leveling or vertical triangulation are then added to the map.

Equipped with this control sheet, on which are shown approximately one trigonometric location per square inch of paper, two or three linear inches of traverse per square inch of paper, and from two to ten elevations per square inch of paper, the topographer walks or rides over all roads and paths and about the margins of lakes, across country, etc., selecting routes so near one another that he may be able to see all portions of the land. If the country is rough he uses an aneroid, setting it at one of the fixed elevations and frequently checking it by others of like kind; with it he is able to determine the crossing of each contour line on his route of travel. In a relatively flat country he uses a hand level, or reads vertical angles from his plane-table and ascertains like facts. Stopping frequently, he sketches in, by means of contour lines, the relief of the country, and corrects as he goes the positions of roads, houses, streams, etc., located by his assistants.

At the close of the field season the temporary employees are discharged and the party chief returns to the office. Here, during the winter season, he completes with great care the drafting of the maps, using the three colored inks selected as standard for this work—black for culture, blue for drainage, and brown for relief. Town and county maps are consulted, correspondence is entered into with postmasters and other local officials and railroad corporations, all compiled material is carefully scrutinized, and the final outlining of political boundaries and the lettering of names are completed.

These original drawings, as they are called, are at once copied by photography or photolithography, so that they may be available for immediate consultation by the public, pending their engraving on copper for final publication.

The cost of the surveys first made, on the scale of 4 miles to the inch, averaged $1 75 a square mile. Those made a few years later, on the scale of about 2 miles to the inch, averaged in cost $4 a square mile, while those made on the scale of approximately 1 mile to the inch, averaged in cost $10 a square mile. During the field season of 1884 a single party mapped over 11,000 square miles. A few years later the output of a single party, on the scale of 2 miles to the inch, was about 3,000 square miles in a season. Today the more refined and detailed maps, on the scale of about 2 miles to the inch, with a contour interval of 100 feet, cost from $7 to $11 a square mile, according to the country, and a single party can rarely map over 600 to 1,000 square miles in a season. On the scale of 1 mile to the inch, a party now maps rarely more than 500 square miles in a season, and the cost of this work varies between $12 and $30 a square mile, according to the nature of the country.

Being in this Institute more particularly interested in the development of the mineral industries of our country, we may ask how this topographical work directly aids the miner. This is easily answered. While the topographical branch of the U. S. Geological Survey is carrying forward the topographical survey of the country, parties being at work in almost every state, the geological branch of the Survey is at work making detailed geological maps of special mining areas. So much is this work appreciated in the United States that it is a regular custom when deposits of valuable minerals are found in any district to at once apply to the Director of the United States Geological Survey, for the construction of a geological map of the area or mining camp in question. Thereupon the Director of the Survey fixes the boundaries of the area to be surveyed and constitutes it a "quadrangle," naming it after some town or village within the area. This quadrangle is bounded by proper north and south and east and west lines, and thus forms a sheet of the geological map of America which is now in course of construction. This having been done, a topographical party at once makes a topographical map of the area on the required scale, and this map, without waiting for the edition to go to press, is given in manuscript form to the geologist and is used by him as the basis of his work in constructing a geological map of the area. In fact, no less than four maps of the area are constructed; one showing the topography of the area; another showing accurately the geology of the area and the position of mineral deposits within it if these exist; while a third is so colored as to emphasize the mineral deposits or any bands or occurrences of rock which are likely to contain them, thus serving as an excellent basis for the work of prospectors. A fourth map shows the structure of the area by presenting a series of transverse sections through it. These four maps are bound together in a stout paper cover and constitute what is known as a "folio." By the aid of the contoured topographical map, all surface workings of the camp can be carried out in the most economical manner, the best location for all railways, &c., can be determined, and the general surface development of the area also worked out was to secure the greatest efficiency at the lowest cost. In the geologically colored sheet the distribution and relative position of the various strata, &c., underlying the area, as well as the outcrops of the mineral deposits, are shown, and from it can be determined the best location for shafts, inclines, drifts, &c., and by it the general development of the underground work can be greatly assisted. Every folio is also accompanied by a sheet of text, which gives a condensed description of the geology of the area in question, written in plain language so that it can be easily understood, while on the inner surface of the cover of the folio there is a brief explanatory treatise of a very general character, and which is identical in all the folios, setting forth the chief geological and topographical terms and explaining the nature of the commoner geological structures. These folios have been found by the mining community of the United States to be of the greatest practical value in the development of the various mining camps, and they have probably done more than anything else to commend the Survey to the mining community of the country.

Such folios would be of inestimable value in the development of the various Canadian mining camps, such as Rossland, those of the Boundary district, or our most recently discovered mineral district about Cobalt, and would commend themselves to Canadians, as those of the American mining camps have to the mining community of the United States. Instead of carrying out such work as this, the Canadian usage has been to send out some unfortunate geologist, with an inexperienced assistant, and to call on him to make a geological map of some complicated area without any topographical basis on which to plot his work. He is then required to furnish a report upon the area, accompanied by a geological map, at once. Such a map however, if it is to be of any value, can only be constructed very slowly, seeing that the geologist is called upon to do, not only his own share of this work but to make a topographical map of the area as well. The geologist, who commands a relatively high salary is thus obliged to spend about three-quarters of the time at his disposal in doing topographical work, which is necessarily of an indifferent quality, and thus has comparatively little time left in a season to devote to that work which properly belongs to his profession and for which he was sent into the field. Such a system can never give satisfactory results, either to the mining community or to the geologist, for it gives the former but little aid and brings discredit on the latter. In the remote and comparatively unexplored regions of northern Canada, where geological work is necessarily of the nature of a reconnaissance, the geological exploration may be carried on at the same time as a rough topographical map is being produced. But in mining areas very different methods must be employed.

There are very few ways in which the Government could render aid to the development of the Dominion more certainly and more wisely than by the organization of a proper topographical survey, and the most convincing evidence of the appreciation which is extended to such work, by people engaged in every species of enterprise which has to do with the development of the resources of a country, can be had by the perusal of Document 136 of the 57th Congress of the United States, entitled "Abstract of Reports, Letters, Clippings, Endorsements, Resolutions, Petitions and Requests, showing the demand for the Topographical Work of the United States Geological Survey," which was published in Washington in the year 1902. This document comprises sixty-two pages of closely printed text, and consists of letters of commendation from all parts of the country, and requests for the construction of topographical maps of special areas by the United States Geological Survey, in order that such maps may be used for the practical development of the resources of the said areas in one direction or another.

The work of surveying Canada when once done would be a permanent asset to the Dominion, and until such a survey is carried out an enormous amount of duplication of effort and waste of money will take place, all of which could be saved by the construction once for all of a proper topographical map of at least the more settled portions of the country.

In concluding the words of the Hon. C. D. Walcott, Director of the United States Geological Survey, in a letter recently received, may be quoted: "Such maps are an essential to the proper development of all of the resources of a State. We have reached the point where officials of railroads, of trolley lines, those interested in highway improvements and in the development of city water supplies and sanitary measures, as in general engineering work, do not think of spending one dollar on field surveys until they have first procured from this office the topographic maps and the records of levels run in connection there-with. We are in daily receipt of testimonials showing the sums of money and the time saved in connection with all preliminary and also of some location surveys for such enterprises. Above all, however, is the topographic map an essential to the study, along modern lines and by best methods, of the mineral researches of a region. The old descriptive methods of discussing the geology of a district have been long abandoned in this country. It has been found that every dollar expended in preparing a mathematically correct base map upon which to map the geologic formations for strike and dip and from which to construct structural sections, is returned manyfold in the value and accuracy of the resulting studies of the geologic formations and their economic possibilities. A good topographic map is just as essential a framework upon which to reproduce a facsimile of the underlying geology, as is the steel frame which modern architects use to form and support a masonry building of modern design."

It is greatly to be desired that the Government of Canada should take steps in the immediate future to provide for the Dominion, that basis for sound industrial development a good Topographical Map.

THE

AMERICAN JOURNAL OF SCIENCE

[FOURTH SERIES.]

ART. XI.—*An Investigation into the Elastic Constants of Rocks, more especially with reference to Cubic Compressibility;* by FRANK D. ADAMS and ERNEST G. COKER.

Introduction.—In a paper published in 1901 an account was given of an investigation into the deformation of marble under varying conditions of heat and pressure.* Under a grant from the Carnegie Institution of Washington to F. D. Adams, the experimental investigation into the behavior of rocks under pressure has been continued in the laboratories of McGill University. As the investigation went forward, however, it was found necessary to follow out several separate lines of research, the results of which it is proposed to present in a series of separate papers.

In the present article a brief résumé is presented of the methods adopted and the results obtained in an investigation into the elastic constants of rocks, more especially with a view to ascertaining the amount of cubic compression which rocks will undergo when submitted to pressure from every side, all rocks of course being subjected to such compression to a greater or less extent, previous to the deformation which they suffer under conditions of differential pressure.

A full presentation of the results of the investigation will be found in a Publication of the Carnegie Institution of Washington, No. [illegible] having the same title as the present paper, and which is now in press, to which the reader is referred for full details of all the measurements carried out in the case of each of the specimens examined, as well as for detailed descriptions, color process photographs and microphotographs of each of the rocks employed. Before the investigation was completed, Dr. Coker was called to the Professorship of Mechanical Engineering in the Finsbury Technical Institute of London, England, and was accordingly obliged to give up the work of the research. His place was taken by Mr. Charles McKergow, Lecturer in Mechanical Engineering in McGill University, but who, immediately on the completion of the work, was appointed to the Professorship of Mechanical Engineering in the University of Virginia. A large number of the very careful measurements of the elastic constants which are presented in the paper were made by Professor McKergow.

---

* Adams, F. D., and Nicolson, J. T. An experimental investigation into the Flow of Marble. Phil. Trans. Royal Soc. London, Series A, vol. cxcv, pp. 363-401.

Methods which may be used in the determination of the Elastic Constants of Materials.

The determination of the cubic compressibility of solid substances is, as above mentioned, beset with serious difficulties. On one hand every direct method which has been suggested presents experimental difficulties which tend to impair its accuracy, while on the other hand the indirect methods are based on assumptions as to the isotropy of the materials, which are not warranted in the case of certain rocks. The indirect methods, however, depending on the theory of elasticity, are capable of considerable variation, and it is of interest to examine them in some detail in order to see whether certain of them at least may not be depended upon to give reliable and satisfactory results.

The determination of the elastic constants of metals has engaged the attention of many physicists and at the present time a large amount of information exists as to the values of these constants for various metals.

It is well known that in homogeneous elastic substances, a simple compression stress causes a lateral strain, which bears a fixed ratio to the compression strain for any particular substance within the limit of elasticity. If then* we call p_x the stress on a plane perpendicular to x in the direction x, and e_x the corresponding strain, then for a direct compression stress p_x there will be a strain in the direction of this stress of amount p_x/E, where E is Young's modulus, and lateral strain of magnitude p_x/mE, where m is the ratio of the longitudinal compression to the lateral extension per unit of length.

If we suppose further that a body is subjected to cubical stress of intensity p_x, we easily see that for small and therefore superposable strains, the cubical strain e_c is

$$e_c = 3p_x \frac{m-2}{mE}$$

---

*See Ewing's Strength of Materials, Chapters I and II.

and since the modulus of compressibility D is the ratio of the stress per unit of area to the cubical strain produced, we have

$$D=\frac{p_0}{e_v}=\frac{1}{3}\,\frac{m}{m-2}\,E$$

Hence if we know E and m we can calculate the value of D.

Further, it is shown in treatises on elasticity that if C is the modulus of shear, then

$$C=\frac{1}{2}\,\frac{m}{m+1}\,E$$

and since C and E are quantities which can be ascertained by experiment, we can from them calculate m and D.

In an important paper by Nagaoka* this latter method has been used to determine the elastic constants of a series of rocks. The value of E was determined by supporting a bar at the ends and measuring the angular change at the support due to a given load applied at the center; the value of E is then obtained by the formula $E=3wl^2/4bd^3\theta$, where l is the length of the bar between the supports, b is the breadth of the bar, d the depth and θ the angular change at the ends for a load W. In order to determine the value of m, a specimen of rectangular section was twisted by a given torque, T, and the amount of the strain measured. It has been shown by St. Venant that for such a case the value of C is given by the formula

$$T=C\theta b^3h\left[\frac{16}{3}-\frac{32^2b^4}{\pi^5}\sum_0^{\infty}\frac{\tan h\,(2n+1)\frac{\pi h}{2b}}{(2n+1)^5}\right]$$

where θ is the angular change, and from this formula values of C were calculated from the observations.

This method appears to be open to some minor objections in that the formula for determining E is based upon a theory of flexure, which although sufficient for many purposes is nevertheless only approximate, and it is well known that values of E obtained by flexure experiments in this manner often differ from the values of E obtained by direct compression experiments by not inconsiderable amounts.

Further, in experiments upon the deflection of beams cut from rocks, it is difficult to obtain consistent readings because of the time effect of the loading, and this difficulty is noticed in the paper cited.

Experiments on the determination of the elastic constants of rocks when subjected to twist were also found to be frequently unsatisfactory owing to the low ultimate shearing value of many rocks.

---

* Elastic Constants of Rocks and the Velocity of Seismic Waves; H. Nagaoka.—Phil. Mag., vol. L, 1900, p. 53.

While a glance at the list of rocks whose elastic constants have been measured by Nagaoka will at once show that most of them are rocks whose elasticity must be of a very imperfect kind, e. g., weathered clay slate, schalstein, tuff, etc.; the method which he has employed for the determination of Young's modulus gives very low results, even in the case of rocks such as marble and granite, where the elasticity might be supposed to be of a high order and comparable to that which these rocks are shown to possess in the case of the types selected for investigation in the present paper. This is seen in the following figures representing the values obtained by him for each of the marbles and granites contained in his list:

Paleozoic Marble:	*E* (Young's modulus).
No. 11A	10,120,000
11B	7,950,000
12A	5,440,000
12B	4,770,000
Granite	
No. 69 (Shodoshima)	6,140,000
68 (Hitachi)	2,853,000
71 (")	2,175,000
56 (")	1,588,000
52 (")	3,265,000

Of these, marble No. 11, if a mean of the two readings be taken, has about the same modulus as the average of those on our list, while No. 12 is very much lower. The highest value given for any granite in Nagaoka's list, viz: No. 39, is somewhat higher than that of the lowest of the granites in our series, that from Stanstead. The other granites examined by Nagaoka have values for *E* assigned to them which are so low that they are comparable only to that of the sandstone in our series. Of the three sandstones included in Nagaoka's list the Izumi sandstone of the Mesozoic has modulus of 1,322,000, while the other two, which belong to the Diluvium, have values for *E* of 587,500 and 583,000 respectively.

And so when an attempt is made to calculate the cubic compression (*D*) from the values given in Nagaoka's list and obtained by his method, it is found that a negative value is actually obtained in about one-third of the rocks which he has examined. His figures, however, were intended chiefly for the purpose of calculating the velocity of the propagation of earthquake shocks.

In consequence of a number of somewhat unsatisfactory results obtained by the writers in some preliminary experiments with this method, as well as the facts with regard to Nagaoka's figures just mentioned, it was decided to adopt a somwhat different method and one which avoided both torsion and flexure and depended simply on strain produced by simple compressive stress. This will be termed the "Method of Simple Compression." Among the possible indirect methods, this seems to be the most satisfactory, since the assumptions necessary in the calculation of compressibility are reduced to a minimum, and the range of stress for which the ratio of stress to strain is practically constant is great.

It was found possible to measure the strain obtained very accurately by means of an apparatus forming part of the equipment of the testing laboratory of McGill University, for the use of which we are indebted to Professor H. T. Bovey. This is an instrument designed by Professor Ewing, and of which a sketch is given in figure 1. In this, *A* is a specimen of the rock gripped by screws passing through a pair of collars, *B*, which are 1·25 inch apart, to which latter metal rods, *C*, are attached. The upper rod carries a glass plate, *D*, with a fine line scratched upon it, the position of which can be adjusted by a screw, *E*, while the lower rod carried a micrometer-microscope, *F*. The upper and lower collars, *B*, are connected by a stud, *G*, the upper one engaging with the conical hole of the swivel piece, *H*, in the lower, and contact is maintained by a spring, *I*, while the weights of the microscope and projecting arms are balanced by lead cylinders, *J*. A buzzer was attached to the upper lead cylinder which when operated caused a slight vibration in the instrument, producing a perfect adjustment as the pressure was applied.

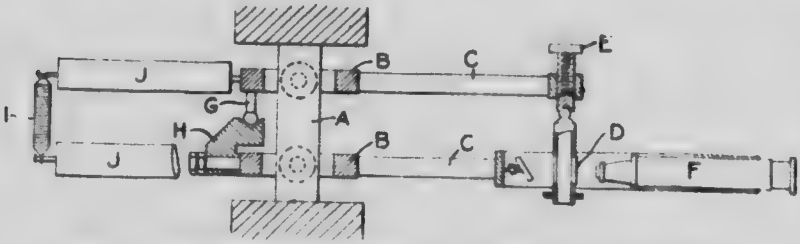

Fig. 1. Instrument for determining the modulus of a simple strain.

The proportions of this instrument were so adjusted that one division on the micrometer scale corresponded to $\frac{1}{250000}$ of an inch, and before using it the instrument was calibrated by aid of a Whitworth measuring machine and was found to be in correct adjustment.

The linear strain perpendicular to the length of the specimen was measured by an instrument which had been designed by E. G. Coker some time previously for experiments on the lateral strains developed in metals.* This is shown in figure 2, and consists of a pair of brass tubes, *B*, *B′*, provided with set screws, *A*, *A′*, for attachment to the specimen, and connected together by a flexible steel plate, *F*, forming the fulcrum. The ends of the tubes near the fulcrum plate are pressed apart by an adjustable spring, *S*, to insure a uniform pressure on the screw points gripping the specimen. On the opposite end of one of the tubes is a spring, *D*, of ebony pressing against a double knife-edge, *K*, seated in a shallow V-notch cut in the end of the other arm. The knife-edge carries an adjustable mirror, *M*, so that if any change in the diameter of the specimen occurs the two tubes move relatively to one another in a horizontal plane, thereby causing the knife-edge mirror to rotate; the rotation of this latter is observed and measured by a telescope and scale placed at a suitable distance.

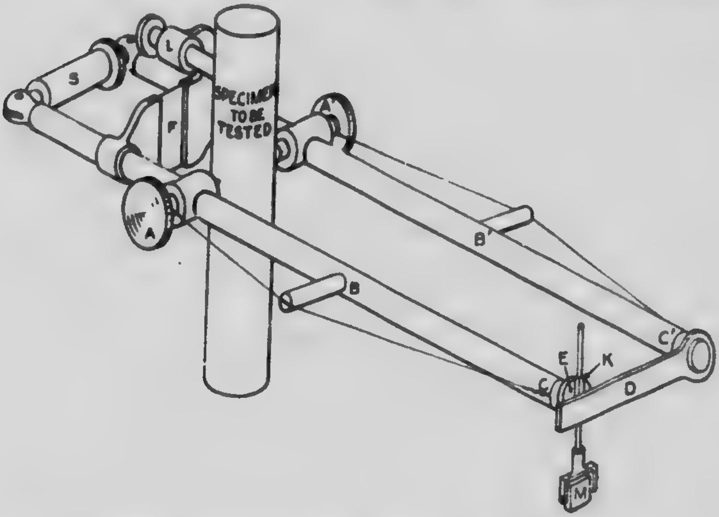

Fig. 2. Perspective view of lateral extensometer.

For convenience in adjustment, there is a screw, *L*, for tilting the apparatus about the axis of the gripping screws, and the tubes, *B*, *B′*, are trussed to prevent vibration. This instrument was calibrated by aid of a Whitworth measuring machine and the scale adjusted so that one division corresponded to one millionth of an inch.

---

* See Proceedings Royal Soc. Edinburgh, Session 1904-5, vol. xxv, pt. vi.

Application of the Method of Simple Compression to the Determination of the Cubic Compressibility of Metals.

The behavior of such metals as wrought iron and steel over a wide range of stress shows that these metals may be considered as almost perfectly elastic. The results of the theory of elastic bodies may therefore be applied in their cases with great confidence.

As a typical example of the behavior of such materials we may consider the deportment of a specimen of wrought iron when subjected to a cycle of compression stresses, commencing at 1,000 pounds and rising to 9,000 pounds, afterwards returning to the original load. The readings obtained for the longitudinal and lateral strains will show in such a case that equal increments or decrements of load produce strains which are very exactly proportional thereto. This is clearly shown in a plot of these readings, where the ordinates represent the total load and the abscissæ represent strains. In both cases the relation of stress to strain is represented by a straight line returning upon itself. Traces which vary but very little from the ideal straight line are given by black Belgian marble, as will be seen on page 114.

Such results afford an arbitrary standard by which can be judged the degree of approximation to perfect elasticity exhibited by other metals and by rocks under similar conditions.

If we now calculate the value of the modulus E for simple compression, since this is the relation of the compression stress p to the strain e, we have

$$p = Ee.$$

If we call A the cross-sectional area of the specimen when stressed by a load P, and x the decrease of length over a measured length L, gripped between the screw points of the measuring apparatus, we obtain

$$E = \frac{PL}{xA}$$

which in the case of a specimen of wrought iron examined for a range of 8,000 pounds, gave a value of 28,100,000, the units being pounds and inches.

The ratio m of the longitudinal strain to the lateral strain in the same case was 3·65, and using the formula

$$D = \frac{1}{3} \cdot \frac{m}{m-2} E$$

we obtain for the modulus of cubical compression (or bulk modulus) D, the value 21,300,000, a constant for the material, the reciprocal of which gives the decrease in volume of 1 cubic inch for 1 pound of pressure.

While certain rocks, such as many of the marbles, have a structure identical with that of wrought iron, most of the rocks constituting the earth's crust are composed of several minerals, and thus rather resemble cast iron in character, the gray variety of this substance being an aggregate of crystals or individuals of the metal iron (wrought iron), graphite, etc.

It will therefore be of interest to ascertain how a specimen of cast iron behaves under compression stress, and how far its elasticity falls short of that which would be exhibited by a perfectly elastic body.

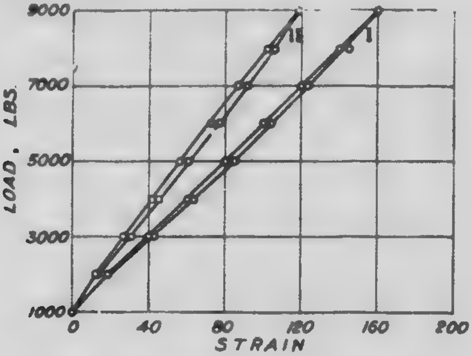

Fig. 3. Cast iron stress strain curves.

For this purpose a fine-grained specimen of somewhat hard cast iron was faced and tested. The stress strain curves are plotted in figure 3. I represents longitudinal compression and II lateral extension.

The behavior of cast iron as exhibited by these experimental results shows a falling away from the theoretical standard of perfect elasticity, but even in the most perfectly elastic bodies there is probably a slight hysteresis effect, so that we are justified in using the results obtained to calculate the modulus of compressibility, if the error introduced thereby is negligible or very small.

It may be pointed out that this method and others of the indirect type have been freely used to obtain values of the bulk modulus for cast iron and metals of like character, and it will be shown that the composite crystalline rocks are very similar to cast iron in their behavior under stress, although generally more perfectly elastic.

Application of the Method of Simple Compression to the Determination of the Compressibility of Rocks.

As has been noted, it was found in the case of marble when subjected to bending stress, that the strain as exhibited by the deflection of a point of the bar increases with the time, and the strength under shear produced by a torque was also so small that a determination of the strain was very difficult to measure.

These difficulties have been noted by Nagaoka,* who states that "Preliminary experiments on granite showed that Hooke's law does not hold even for very small flexure and torsion, and that the after effect is very considerable when the prism is sufficiently loaded or twisted; the deviation from the direct proportionality between strain and stress was incomparably great as compared with that observed in common metals. This must be chiefly due to the low limit of elasticity, so that it is necessary to experiment only within narrow limits of loading and twisting. These limits are widely different for different specimens of rocks, and the modulus of elasticity as well as that of rigidity was always determined with such stresses as will approximately produce strains proportional to them. The deviation from Hooke's law was prominent in certain specimens of sandstone, and it was more marked in torsion than in flexure experiments; in certain rocks it is indeed doubtful if anything like a proportionality between stress and strain can be found, even for extremely small change of shape: on releasing these rocks from stress the return toward the former state is extremely small, showing that the elasticity of the rocks is of very inferior order."

If, however, the rock be subjected to direct compression, strains in which the time effect is small, and the lag of the strain is also small, are almost invariably obtained. This is especially the case if, before the actual experiment is carried out, the material be several times subjected to a range of stresses at least as great as those employed in the experiment itself. This preliminary stressing brings the material to a "state of ease," and is also commonly adopted when the elastic constants of metals are determined.

It is evident therefore that this Direct Compression Method may with confidence be applied to the measurement of the cubic compression of rocks, although as mentioned below the accuracy of the results obtained will differ with different classes of rocks. If the rocks be massive, compact and crystalline (or glassy), the method can be safely employed and good results will be obtained. If, on the other hand, the rock is schistose, porous or loosely coherent, the method will, from the nature of the case, be very much less satisfactory.

---

* Elastic Constants of Rocks and the Velocity of Seismic Waves; Nagaoka. H.—Phil. Mag., vol. L, 1900, p. 58.

The plutonic igneous rocks as a class most nearly resemble the metals in structure, being holocrystalline and massive, and therefore present the nearest approach among rocks to perfectly elastic bodies: they are therefore a class of rocks to which this method is especially applicable. It fortunately happens that they also form a class of rocks a knowledge of whose compressibility is of special importance for the elucidation of geological problems, constituting as they do the greater part of the earth's crust.

A second class of rocks which are comparable to them in their approach to perfect elasticity, comprises the marbles and certain limestones.

A series of sixteen typical rocks representative of these two classes were accordingly selected for measurement: under the first class a number of granites were chosen as representing the acid plutonic rocks, and a number of types of the gabbro-essexite series were selected as representing the basic plutonic rocks. In all cases great care was taken to choose the most homogeneous and massive rocks of each series, and to secure test pieces free from all flaws and cracks. As representing the second class a number of typical marbles and limestones, also perfectly massive in character, were selected. For purposes of comparison, or contrast, a sandstone was added to the list as being a rock which, on account of its more or less porous nature, could hardly be expected to yield satisfactory results by this method.

An examination of the stress strain curves of these 16 rocks, omitting the sandstone, shows that on the average they possess a rather more perfect elasticity and exhibit less hysteresis than cast iron. Some of them, as for instance, the Baveno granite, the nepheline syenite, the diabase and the black Belgian marble, show much better curves, approximating in fact to the straight lines given by wrought iron, which may be considered for our present purpose as expressing perfect elasticity.

The close approximation to perfect elasticity is shown by the return of the curve to its initial or starting point, and the amount of the hysteresis is shown by the width of the loop.

The width of this hysteresis (or lag) curve or loop indicates the amount of the divergence from Hooke's law which the material exhibits—this law being that the stress and strain are *directly* proportional. When the curve is narrow, as it is in all cases except the Stanstead granite and the sandstone, the divergence from Hooke's law is not great enough to seriously affect the result.

The rocks, therefore, with these exceptions, fulfil the conditions of elasticity necessary to the successful application of the method. In these two cases the results are less certain owing to the greater hysteresis of the rock.

It might at first sight appear that while the method employed is theoretically perfect as applied to the measurement of the compressibility of vitreous rocks and of very fine-grained crystalline rocks, a considerable error might be introduced when the rocks are coarser in grain. In the case of all the common crystalline rocks, the individual grains of which the rock is composed are anisotropic, that is they have different moduli of elasticity in different directions. In massive rocks such as those investigated, however, these grains occur in the rock with an absolutely irregular orientation and would in the case of a fine-grained rock mutually compensate for one another in any transverse line along which the expansion of the rock under compression might be measured. If, however, the rock were coarser in grain, fewer individual crystals would be found in any transverse line of section, and there might possibly in this way be lack of compensation, as the rock in one section might be composed of grains whose axis of greater elasticity approximated on an average more nearly to the direction of measurement than in other sections. If such were really the case, there should be in these coarser-grained rocks an exceptionally great variation in the readings obtained from different specimens of the same rock, as well as from the different sections in the same specimen.

But such is not the case, as will be seen by an examination of the figures in the accompanying table. These represent the results obtained from ten measurements of the compressibility of Baveno granite, which is coarse in grain, and ten of Sudbury diabase, which is very fine in grain, together with eight measurements of Tennessee limestone, which is rather coarse grain, and seven of plate glass. They were made in each case on two or more specimens cut from the same mass, and the measurements of the expansion were made on several different planes through each, so that in every case the measurement was effected in a different line through the rock, all of these, however, of course being at right angles to the direction of the compressive stress and lying in the medial plane of the column.

	Maximum.	Minimum.	Difference.
Baveno granite (coarse), 10 trials................	4,880,000	4,380,000	500,000
Sudbury diabase (very fine), 10 trials...............	11,170,000	9,655,000	1,515,000
Plate glass, 13 trials	6,930,000	6,020,000	910,000
Tennessee marble (rather coarse), 7 trials........	6,130,000	5,770,000	360,000

It will thus be seen that there is no correspondence between the coarseness of grain and the magnitude of the variations in the readings obtained. The differences in glass, which is an isotropic material in which the elasticity is equal in all directions, are greater than in the Tennessee marble, which is rather coarse in grain, and in Baveno granite, which is the coarsest rock of the set. The greatest differences obtained are those found in the finest-grained rock in the series, viz., the Sudbury diabase.

It is evident therefore that the different moduli of elasticity of the constituent grains of a rock do not introduce any perceptible error in measurements made by this method, when a column an inch in diameter is employed, and when the rocks are not coarser in grain than the Baveno granite. In fact, when surrounded on all sides by other grains, no individual grain can expand freely, as it would if subjected to compression unhampered by any surrounding medium, and thus the anisotropic character of the individual grains produces but little effect on the elasticity of the rock as a whole.

These experiments also show that in the case of rocks composed of several minerals, it makes no perceptible difference whether the points of attachment of the instrument are embedded in the grains of one mineral or of another. The chief source of error, and the one to which the variations observed are for the most part to be attributed, seems to be a mechanical one, viz., the difficulty of getting an ideal contact between these points of attachment and the specimen, especially in view of the extremely small dimensions of the movement to be measured.

The question of the influence of temperature on the elasticity and compressibility of rocks is of course one which has an important bearing on certain problems of geophysics. The only investigation of this subject, so far as can be ascertained, consists of a few preliminary experiments by Nagaoka and Kusakabe.* In these the torsion method was employed, and the experiments were carried out on a single rock, viz., sandstone. This rock, as has already been mentioned, being porous and stratified in character, is a material whose elastic properties are far from ideal. The results are summed up by the authors in the following words: "Preliminary experiments with sandstone show that the modulus of elasticity is much affected by the variation of temperature, i. e., about 0·5 per cent. per degree. It does not, however, necessarily diminish with the increase of temperature where the temperature is low, i. e., it is maximum about 9° C."

As has been shown, however, the values for the elastic constants obtained by this torsion and bending method have yielded results which cannot in all cases be correct, and which differ very considerably from those obtained by the much more direct and simple method which has been employed in the present paper. These results bearing on the variation of elasticity induced by changes of temperature, especially in view of the fact that they are stated by the investigators to be "preliminary," can as yet hardly be taken as of general application to all rocks, even if correct for the specimen of sandstone examined.[*]

---

* Modulus of Elasticity of Rocks, and Velocities of Seismic Waves, Publications of the Earthquake Investigation Committee, No. 17, Tokyo, 1904, p. 43.

In our own investigations the laboratory was maintained at a temperature of from 63° to 68° F. (17·2° C. to 20° C.), and a thorough investigation into the effect of temperature was not undertaken, as this would be very difficult to carry out when employing the method of direct compression used, the difficulty consisting in heating the specimen itself without in any way affecting the measuring apparatus attached to it.

It seemed, however, possible to ascertain whether any serious change in the elastic constants of the massive crystalline rocks employed in the present investigation would result from a moderate change of temperature. For purposes of trial the rock selected was the Sudbury diabase, a typical fine-grained plutonic rock. A column of it was placed by Mr. McKergow in a small testing machine, having a capacity of 50 tons, and the temperature of the room in which the machine was set up having been lowered to +10° F. a cycle of compression readings was taken in the usual way adopted when Young's modulus is to be determined. The temperature of the room was then raised by about 10° and another cycle of readings was taken. It was then raised another 10° and a third series of readings was obtained, and so on through successive stages of 10° until the normal temperature of the room (about 65° F.) was reached. The initial reading of the instrument before the application of pressure was of course different in each case, owing to the expansion of the rock which followed from heating. These initial points were plotted on a line, and the results obtained when this specimen was subjected to a certain maximum load, together with the increase of temperature at each stage, were plotted on a second line. If the compression was greater at 65° than at 10° for the same load, these two lines should have diverged, but as a matter of fact they were practically parallel. The differences between the readings given by the same load at different temperatures were no greater than those obtained by different measurements under the same load at the same temperature. The conclusion therefore seems to be indicated that a change of temperature made no perceptible difference within the range of temperatures employed, although a difference of 0·5 per cent. for each degree centigrade would mean a difference of about 25 per cent in range of temperature employed by Mr McKergow.

---

[*] In two very interesting papers, received while the present paper was in press (Modulus of Elasticity of Rocks and some inferences relating to Seismology, Journal of College of Science, Imperial University of Japan, vol. xx, article 9, 1905; and Kinetic Measurements of the Modulus of Elasticity, etc., ditto, vol. xx, article 10, 1905), Kusakabe shows that the presence of moisture in a rock has a very marked influence in diminishing its modulus of elasticity. This influence, he has ascertained, is very much more pronounced in the case of porous rocks—such as sandstone—which absorb a large quantity of water, than in the case of the compact crystalline rocks. He suggests that the effect, which in the paper above mentioned was attributed to heat, may really be due to the presence of moisture. He is now engaged in investigating the influence of moisture combined with a high temperature upon the modulus of elasticity of various rocks.

While therefore this experiment cannot be considered as supplying accurate information concerning the effect produced by a rise in temperature on the elastic constants of rocks, for the instruments themselves are in some measure affected by the same changes of temperature, yet they serve to show that in the case of the massive crystalline rocks, the influence of temperature is probably not very great.

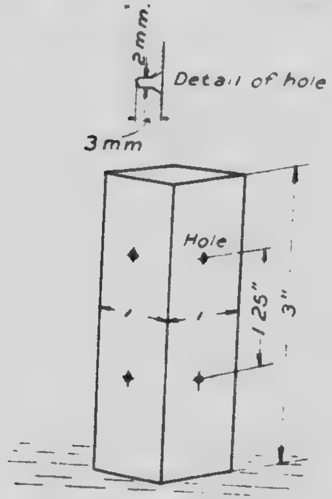

Fig. 4. Square test specimen of rock.

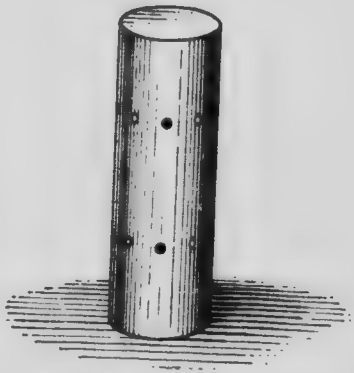

Fig. 5. Round test specimen.

The Method of Measurements.

In carrying out the measurements, prisms of the rock 1 inch square and 3 inches long were usually employed (see fig. 4). These were cut and ground with smooth faces but were not polished. In these two small round holes were drilled in the medial line of each vertical face for the purpose of attaching the instrument, when Young's modulus was to be measured. These holes were made by means of a small diamond drill and were perfectly round and smooth. They were each 0·05 to

0·08 inch in diameter and 0·125 inch deep and 1·25 inches apart, lying at equal distances above and below the center of the prisms. These holes were chamfered at the outer end as shown in figure 4, and were found to afford the most perfect attachment which could be secured for the points of the instrument.

By means of these prisms two sets of measurements of the vertical compression could be made with each prism, by attaching the instrument first to one pair of opposite faces and then to the other.

In some cases round columns were used (see fig. 5). These were approximately 1 inch in diameter and 3 inches in length. With these it was possible to make four sets of measurements in compression with each column, by drilling eight pairs of holes as above described, whose plains intersected at angles of 45° instead of 90° as in the square prisms.

It was of course necessary in every case, whether prisms or columns were employed, to exercise great care to have the end of the test pieces very carefully faced and absolutely parallel to one another. Before the actual measurements were made, the rock in every case was brought to a "state of ease" in the manner already described.

The pressure was applied in most cases by a 100-ton Wickstead testing machine, which was so carefully adjusted that it was sensitive to a load of 4 pounds.

The specimen having been placed in the testing machine and brought to a state of ease was then after careful adjustment submitted to loads increasing in successive stages of 1,000 pounds until the limit of safety had been reached, when the load was reduced successively by the same amounts, accurate readings being taken at each increment and decrement of load. The maximum load employed in the case of most rocks was 9,000 pounds, equivalent to from 9,000 pounds to about 11,500 pounds per square inch according to whether a square or round prism was employed. In the case, however, of some of the stronger rocks a load of as much as 15,000 pounds per square inch was employed.

In the determination of the lateral strain, which was made upon the same columns as those used for measuring the vertical compression, care was taken that the theoretical conditions were realized, and that the material was free to expand laterally, as otherwise the values obtained for the lateral extension would be inaccurate. In all cases, therefore, the measuring apparatus was set as nearly as possible upon the central section of the test piece, and the ends of the specimen, after being ground smooth, were coated with a thin film of oil, so that the polished pressure plates of the machine would have as little tendency as possible to prevent freedom of lateral expansion.

It was found that these precautions being observed, the expansion at the ends of the column was practically as great as at the center where the measurement was taken; the difference being so small that no serious discrepancy was introduced.

In a number of cases accurate measurements were taken during the successive cycles of loading and unloading to which the specimen was subjected in order to bring it to a state of rest. These were recorded in the case of the Baveno granite and the Stanstead granite and serve to show how the hysteresis of the rock may be reduced to a minimum by subjecting the test-piece to this treatment. The measurements of each cycle usually occupied from ten to fifteen minutes.

In the case of a majority of the rocks investigated, a number of prisms or columns cut from the same block of rock were measured in order to ascertain whether different test pieces would give identical readings. It was found as a result of these investigations that the differences between the different specimens were no greater than those which were obtained by measuring the same specimens with the instrument attached to different sides.

In the case, however, of the Quincy granite, test pieces from two different blocks of the rock were prepared, and it was found that while the several measurements made on each test piece agreed among themselves, there was a distinct divergence in the elastic constants of the two specimens of the rock. This was probably due to a difference in composition, as the two rocks differed somewhat in appearance.

In the case of the green gabbro from New Glasgow, the results obtained by measurements made upon different parts of the same prism were discordant for reasons which will be pointed out and which were dependent upon the structure of the rock.

Fifty-five columns of rock, nineteen of glass, and two of iron were employed in this investigation and every precaution was taken to insure the attainment of accurate results. The rocks in all cases were air-dry, having been allowed to remain in the laboratory for several weeks after they had been cut, before the measurements were made.

In the accompanying tables the following elastic constants are given:

E = Young's Modulus, i. e., the quotient of the longitudinal stress by the longitudinal compression.

σ = Poisson's Ratio. This is the reciprocal of m.

D = Modulus of Cubic Compression $= \frac{1}{3}\left(\frac{m}{m-2}\right)E$. The reciprocal of this gives the decrease in volume of a cubic inch of the material for a pressure of 1 pound per square inch applied on every side.

C = Modulus of Shear $= \frac{1}{2}\left(\frac{m}{m+1}\right)E$, which is the quotient of torsional stress to torsional strain.

m = The ratio of longitudinal compression to lateral extension per unit of length.

E and m are measured directly; the other values are calculated from them.

These values in the case of each rock are given in inch and pound units, and the results are summarized in a general table on page 121. The measurements were made in these units on account of the fact that the testing machine employed was graduated to read pounds.

For purposes of comparison, however, this latter table has been recalculated in C.G.S. units, and the results are set forth in the second table, to be found on page 121.

In the case of metal, Poisson's ratio is generally arrived at by stretching the bar and determining the value of the longitudinal extension divided by the lateral contraction. In case of rocks the tensile strength being low and the materials being brittle, it is more convenient and more accurate to make the determination by compressing a short bar or column, and determining the value of the longitudinal compression divided by the lateral expansion. This gives the value designated as m, of which Poisson's ratio is the reciprocal. Theoretically one method is as accurate as the other.

In each table the first transverse line designates the specimen employed as *a*, *b*, *c*, or *d*.

The second line gives the diameter of the specimen, which is often slightly different in the two directions. The length of the column in all cases was about three inches, but this is not stated in the table, as the compression is not measured on the total length of the column, but on the length of that portion of it which lies between the points of attachment of the instrument.

The third line gives the area, which is approximately one square inch in the case of a square prism and three-quarters of a square inch in the case of a round column.

The fourth transverse line contains the letters *U* or *P*, which designate the two diameters of the column when two measurements were made on the same square prism; these two directions being always at right angles to one another. In the case of round columns on which measurements were frequently made in several planes, these are designated as "first holes," "second holes," etc.

In the four succeeding lines the four elastic constants E, σ, D and C are given as determined by each measurement.

The sixteen rocks whose elastic constants were determined are enumerated in the following list:

Marbles and Limestones.

1. Black Belgian marble; an extremely fine-grained and massive black marble, largely used for ornamental purposes and known in trade as "Belgian Black".

2. White marble, Carrara, Italy; a typical fine-grained saccharoidal marble.

3. White marble, Vermont, U. S. A.; identical in appearance with the last.

4. Pink marble, Tennessee, U. S. A.; a highly metamorphosed coralline limestone which has been converted into a marble; largely used for purposes of construction and known as "Pink Tennessee."

5. Trenton limestone, Montreal, Canada; a highly fossiliferous variety, free from any signs of stratification, taken from a heavy bed in the Mile End quarries at this place; used extensively as a building stone.

Granites.

6. Granite, Baveno, Italy; a typical biotite granite of medium grain.

7. Granite, Peterhead, Scotland; a typical, rather coarse-grained, biotite granite.

8. Granite, Lily Lake, New Brunswick, Canada; closely resembles No. 7.

9. Granite, Westerly, Rhode Island, U. S. A.; a typical, very fine-grained reddish biotite granite.

10. Granite, Quincy, Massachusetts, U. S. A.; a rather coarse-grained hornblende pyroxene granite.

11. Granite, Stanstead, Quebec, Canada; a rather fine-grained muscovite biotite granite; the mica is relatively more abundant than in either the Peterhead or the Westerly granites.

Nepheline Syenite.

12. Nepheline syenite, Montreal, Canada; a typical fine-grained massive hornblende nepheline syenite.

Basic Plutonic Rocks.

13. Anorthosite, New Glasgow, Quebec, Canada; a rock composed of plagioclase with a subordinate amount of pale green augite and green hornblende; it is fine in grain and very tough, being used for paving sets in the city of Montreal.

14. Essexite, Mount Johnson, Quebec, Canada; a typical essexite, massive and uniform in character, composed of plagioclase, nepheline, augite, hornblende and biotite; used extensively as a building stone and for monuments.

15. Gabbro, New Glasgow, Quebec, Canada; this rock is deep green in color and occurs in the form of a large dike cutting the anorthosite (No. 13); composed of augite, hornblende and plagioclase, the two former minerals preponderating largely. It

shows a distinct parallelism in the arrangement of the constituents, to which may be attributed a considerable divergence in the results obtained in the different measurements of the elastic constants.

16. Olivine diabase, near Sudbury, Ontario, Canada ; a typical fine-grained, perfectly massive olivine diabase, occurring as a large dike cutting rocks of Huronian age.

Sandstones.

17. Sandstone, Cleveland, Ohio, U. S. A., a fine, even-grained yellowish sandstone, extensively used for building purposes. The bedding is marked by a slight variation in the color of the different beds. The prism used in the measurement of the elastic constants was cut from a single bed and was taken parallel to the plane of bedding.

A summary of the results obtained in the measurement of the elastic constants of these rocks is given on p. 121. The extended results are here presented in the case of three rocks only, which may serve as representatives of the others.

Carrara Marble :

Three specimens of the rock were used in measuring the elastic constants, two squares prisms (a and b) and a round column (c). Two sets of measurements were made on both b and c, the instrument as usual being affixed to the specimens in two positions at right angles to one another in each case. In this way five sets of measurements were made. The results are set forth in the following table :

Carrara Marble

No.	*a*	*b*	*b*	*c*	*c*
Size	1·032×1·035	1·017×1·016	....	·985	·985
Area	1·07	1 033	1·033	·762	·762
E	8,120,000	7,800,000	8,055,000	8,210,000	8,045,000
σ	·281	·274	·273	·275	·269
D	6,170,000	5,750,000	5,920,000	6,100,000	5,790,000
C	3,170,000	3,060,000	3,160,000	3,210,000	3,170,000

The averages of the results obtained for the respective constants are as follows :—

E=8,046,000; σ=0·2744; D=5,946,000; C=3,154,000.

The difference between the highest and lowest determinations of D is 420,000 pounds.

Figure 6 shows the stress strain curves plotted from the results obtained from specimen a, the ordinates representing the load (stress) and the abscissae the amount of the strain. I represents in all cases the longitudinal compression and II the lateral extension. The hysteresis is greater than in the case of the black Belgian marble (fig. 7), but about the same in amount as shown by the Vermont marble and the Trenton limestone from Montreal.

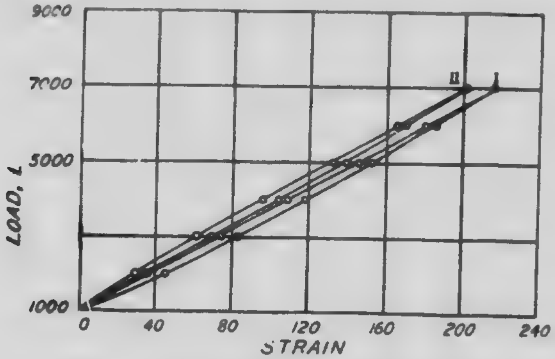

Fig. 6. Carrara Marble, stress strain curves.

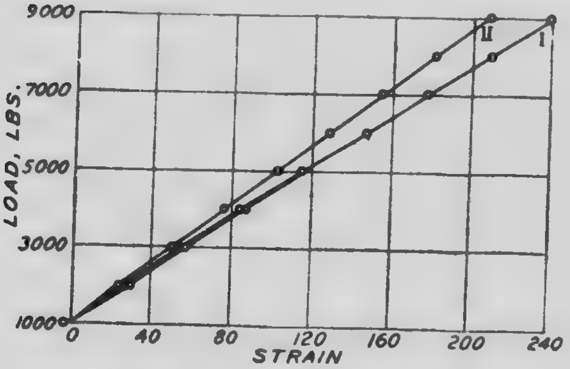

Fig. 7. Black Belgian marble, stress strain curves.

Granite, Westerly, Rhode Island, U. S. A.

Four test pieces were used in measuring the elastic constants, viz: two square prisms, *a* and *b*, and two round columns, *c* and *d*. Two sets of determinations were made on each of the first three specimens, the instruments being attached to different pairs of sides in each case, and four sets of determinations were made on specimen *d* in planes making angles at 45° with one another. The results are given in the following table.

The averages of the values obtained are as follows:

$E = 7,394,500$; $\sigma = \cdot 2105$;
$D = 4,397,500$; $C = 3,019,700$

The difference between the highest and lowest values in the four determinations of D on specimen d was only 280,000.

Of the other columns a gave on an average somewhat lower and b somewhat higher results.

The stress strain curves obtained from specimen a are shown in figure 8. The hysteresis is greater than that shown in the case of any of the other granites except that from Stanstead.

In figure 9 the stress strain curves for the Peterhead granite, and in figure 10 those for the nepheline syenite are shown for purposes of comparison.

Olivine Diabase, near Sudbury, Ontario, Canada.

Four test pieces were used in determining the elastic constants of the rock, viz: three round columns and one nearly square prism. They are designated as a, b, c and d. The three round columns were cut out of a block of the diabase by means of an annular diamond drill. Two measurements were made on each of these in planes at right angles to one another, in each case, while four measurements were made on the prism d using two pairs of faces. In this way ten complete sets of measurements were made for the elastic constants of this diabase.

The values obtained are given in the tables on p. 117.

As will be seen, the values obtained for D in this rock are considerably higher than those yielded by any other rock of the series examined. In the six independent measurements carried out on the first three specimens, the difference between the highest and lowest values for D amounted to 880,000 pounds, while on the four measurements made on specimen d there is a rather greater difference amounting to 845,000 pounds.

Granite, Westerly, Rhode Island, U. S. A.

No.	*a*	*a*	*b*	*b*	*c*	*c*	*d*	*d*	*d*	*d*
Size	1·008 × 1·002	....	·981 × ·929	...	·975	....	·975	....	....	....
Area	1·01	....	·91	....	·75	....	·75	....	....	....
Side	*U.*	*P.*	*U.*	*P.*	1st holes	2d holes	1st holes	2d holes	3d holes	4th holes
E	7,180,000	7,090,000	7,625,000	7,745,000	7,670,000	7,335,000	7,575,000	7,335,000	7,170,000	7,250,000
σ	·21	·1985	·241	·214	....	....	·225	·223	·2225	·222
D	4,110,000	3,950,000	4,925,000	4,515,000	....	....	4,600,000	4,420,000	4,320,000	4,340,000
C	2,970,000	2,960,000	3,070,000	3,185,000	....	....	3,090,000	2,980,000	2,940,000	2,961,000

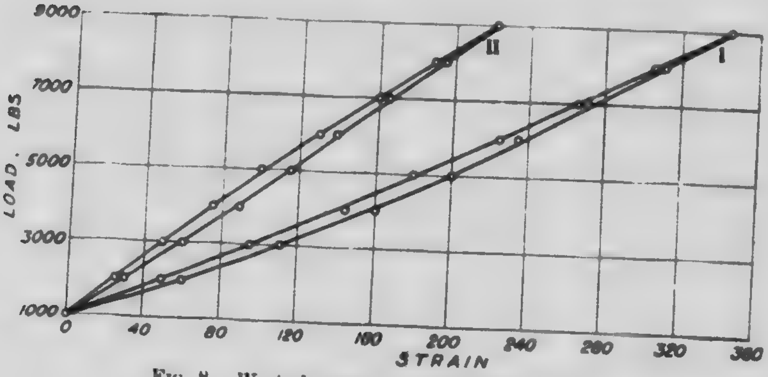

FIG. 8. Westerly granite, stress strain curves.

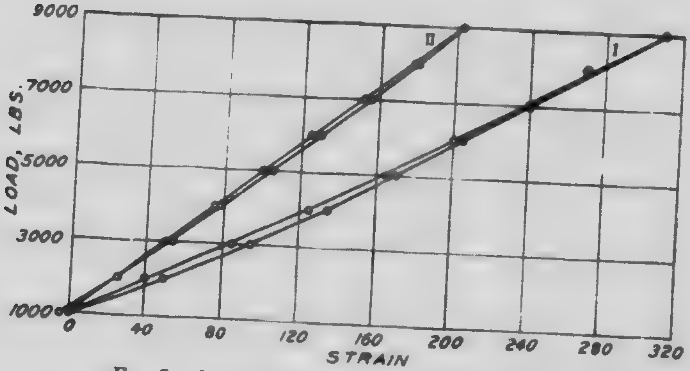

FIG. 9. Peterhead granite, stress strain curves.

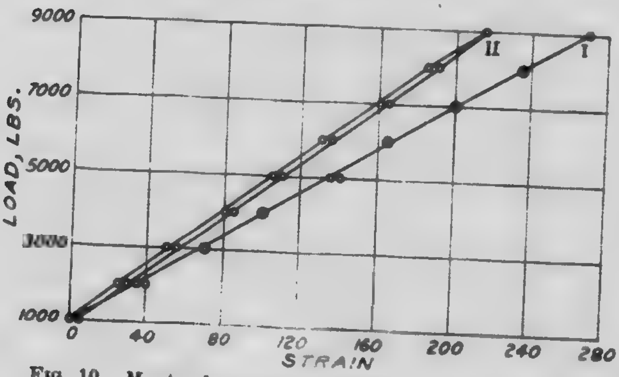

FIG. 10. Montreal nepheline syenite, stress strain curves.

The averages of the determinations made on each of the columns are as follows:

	E	*D*	*σ*	*C*
a =	13,515,000	10,400,000	0·2838	5,270,000
b =	14,170,000	10,945,000	·2840	5,525,000
c =	14,170,000	11,085,000	·2870	5,505,000
d =	13,197,750	10,076,000	·2812	5,155,000
Av.	13,763,187	10,626,500	·2840	5,363,750

The stress strain curves given by this rock are shown in figure 11. As will be seen from these curves, in its approach to perfect elasticity the rock is comparable to plate glass.

The Elastic Constants of Glass.

As in geophysical speculations, the earth in respect to its rigidity and compressibility is often compared to a globe of glass, it seemed advisable to determine as accurately as possible the elastic constants of glass, for the purpose of comparing them with the results obtained in the case of the various rocks considered in this paper, employing the same methods and carrying out the work under exactly the same conditions. This material lends itself excellently to this method of measuring these constants, provided the glass is free from all irregularities in its substance and is isotropic in character. The first difficulty experienced was that of obtaining such a glass. At the outset it was thought that thick glass rods such as are used for various purposes in the chemical and physical laboratory might be employed, but although several lots of the purest variety of this material were procured, the glass constituting it was found in all cases to contain minute air bubbles, and when examined between crossed nicols in polarized light, showed brilliant colors—red, yellow and blue. This indicated a state of marked tension in the glass, evidently due to the rod having been drawn when the glass was in a viscous state, which was also shown by the circular arrangement of the little bubbles in the rod, following the direction of its surface. Short lengths of this rod moreover when tested in compression, so soon as the maximum load had

Olivine Diabase, Sudbury, Province of Ontario, Canada.

	a	*a*	*b*	*b*	*c*	*c*	*d*	*d*	*d*	*d*
Size	·981	·981	·983	·983	·983	·983	1·000 × ·864	...	...	...
Area	·756	·756	·758	·758	·758	·758	·864	·864	·864	·864
E	13,250,000	13,780,000	14,020,000	14,320,000	14,020,000	14,320,000	13,150,000	13,330,000	13,450,000	12,860,000
σ	·2865	·281	·291	·277	·291	·283	·276	·285	·287	·277
D	10,340,000	10,460,000	11,170,000	10,720,000	11,170,000	11,000,000	9,810,000	10,340,000	10,500,000	9,655,000
C	5,160,000	5,380,000	5,430,000	5,620,000	5,430,000	5,580,000	5,170,000	5,200,000	5,230,000	5,020,000

Plate Glass.

No	a	b	c	d	e	f	g
Size	0·9855×1·0205	0·9865×1·0055	0·981×1·0135	1·016×1·008	1·0215×0·9955	1·022×1·0025	1·025×0·994
Area	1·0057	0·992	·994	1·024	1·017	1·024	1·016
E	10,350,000	10,950,000	10,480,000	10,380,000	10,450,000	10,380,000	10,450,000
	10,590,000	10,500,000	10,350,000	10,380,000	10,930,000	10,600,000	10,230,000
σ	0·2281	0·236	0·226	0·233	0·221	0·229	0·216
	0·228	0·2341	0·235	0·227	0·23	0·225	0·215
D	6,370,000	6,930,000	6,460,000	6,480,000	6,380,000	6,370,000	6,140,000
	6,480,000	6,580,000	6,520,000	6,350,000	6,760,000	6,430,000	6,020,000
C	4,220,000	4,440,000	4,280,000	4,210,000	4,280,000	4,220,000	4,300,000
	4,310,000	4,250,000	4,190,000	4,230,000	4,440,000	4,330,000	4,360,000

been exceeded, instead of splitting from top to bottom, broke as if composed of a series of rudely concentric shells.

After a prolonged search for isotropic glass in masses of sufficient size to measure the elastic constants, it was found that plate glass answered the requirements. A piece of one inch plate glass, made in Great Britain, was accordingly secured and was cut into strips an inch wide, and these again into three inch lengths. The square prisms thus produced were then properly faced and polished. The glass was found to be absolutely free from all flaws and impurities, and when examined between crossed nicols, the prisms although an inch thick, showed in one direction at right angles to vertical axis absolute blackness throughout a complete revolution, while in the other direction at right angles to this there was during a revolution an alternation of blackness with a pale grayish illumination. This change was so slight that considering the thickness of the glass and the sensitiveness of the test, the material may be considered to be practically free from internal tension, and to be isotropic in character.

In order to get a good average and to eliminate chance errors so far as possible, seven of these prisms were taken, and two complete sets of determinations were made on each of them, using in every case different pairs of faces. Fourteen determinations were thus made of each of the elastic constants. The figures obtained are set forth in the table on p. 118.

In this table a complete series of values obtained from each specimen are given in double rows. When the average of all these results is taken, the values obtained for the several constants of plate glass are as follows:

$$E = 10{,}500{,}000\,; \quad \sigma = 0{\cdot}2273\,; \quad D = 6{,}448{,}000\,; \quad C = 4{,}290{,}000$$

The stress-strain curves given by one of the prisms is shown in figure 12. In this figure I represents longitudinal compression and II lateral extension.

Determinations of the cubic compressibility of glass, *D*, have been made by other observers using various methods.

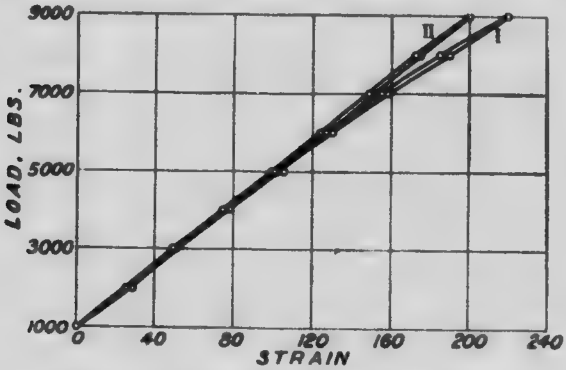

Fig. 11. Sudbury Diabase, stress strain curves.

The results go to show that different varieties of glass vary considerably in their compressibility. These determinations may be tabulated as follows:*

Everett	5,074,600 to 6,379,400 (C.G.S. $= 3{\cdot}5$ to $4{\cdot}4 \times 10^{11}$)
Amagat-common glass	6,745,000 (·000002181 per atmosphere)
Amagat-crystal glass	6,112,300 (·000002405 " ")
Tait	5,657,700 (·0000026 " ")

As will be seen, the figures obtained for plate glass in the present investigation lie a little above the average of the various values here given, and are nearly those of the highest value obtained by Everett.

---

* See Everett, Illustrations of the C.G.S. System of Units with tables of Physical Constants. Macmillan & Co., 1902, pp. 60 to 64. The figures there expressed in various units have been here recalculated into inch-pound values.

Summary of Results.

In the table on the following page a summary is shown of the average values obtained for E, σ, C and D in the case of all the rocks examined in this investigation. Together with these are placed for purposes of comparison the results obtained for these constants in the case of wrought iron, cast iron and glass. In the second table these values are again presented, recalculated into C.G.S. units.

The rocks fall naturally into three groups differing from one another in compressibility, but the several members of each group agreeing fairly closely among themselves.

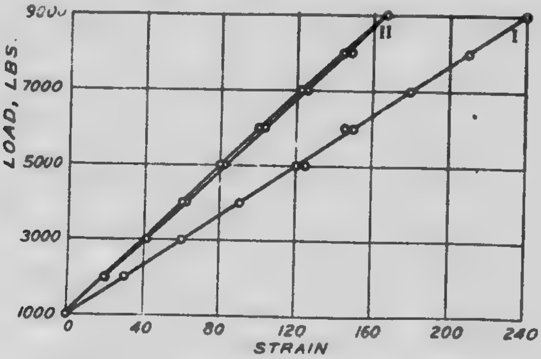

FIG. 12. Plate glass, stress-strain curves.

These three groups show a corresponding difference in composition.

The first group consists of the marbles and limestones. These have an average value for D of 6,345,000. One of these, however, the Black Belgian marble, which is very much finer in grain than the others, and breaks almost like a piece of glass, has a very much higher value for D than that possessed by the other rocks, which among themselves are nearly identical. If we omit this Belgian marble, the average of D for the other limestones and marbles is 5,855,000.

The second group comprises the granites. These again show a close agreement of values among themselves, except in the case of the Stanstead granite, which rock as already mentioned shows a defective elasticity. The average value of D for the granites is 4,399,000.

The third group embraces the basic intrusives (gabbro, anorthosite, essexite and diabase). These show greater differences, but have an average value for D of 8,825,000. The nepheline syenite, although higher in silica and therefore properly speaking an acid rock, in its freedom from quartz and its richness in feldspar (although the feldspar is largely orthoclase instead of plagioclase), in mineralogical composition belongs with these basic rocks rather than with the granites. It also approaches the essexite most nearly in its compressibility.

ELASTIC CONSTANTS OF ROCKS.

Summary of Results (average) expressed in Inch-Pound Units.

	E	σ	C	$D=\frac{1}{3}\left(\frac{m}{m-2}\right)E$
Wrought iron	28,100,000	0·2800	11,000,000	21,300,000
Cast iron	15,000,000	0·2500	6,000,000	10,000,000
Black Belgian marble	11,070,000	0·2780	4,330,000	8,303,000
Carrara marble	8,046,000	0·2744	3,154,000	5,946,000
Vermont marble	7,592,000	0·2630	3,000,000	5,341,000
Tennessee marble	9,006,000	0·2513	3,607,000	5.967,000
Montreal limestone	9,205,000	0·2522	3,636,000	6,167,500
Baveno granite	6,833,000	0·2528	2,724,800	4,604,000
Peterhead granite	8,295,000	0·2112	3,399,000	4,792,000
Lily Lake granite	8,165,000	0·1982	3,380,000	4,517,500
Westerly granite	7,394,500	0·2195	3,019,700	4,397,500
Quincy granite (1)	6,747,000	0·2152	2,781,600	3,984,000
Quincy granite (2)	8,247,500	0·1977	3,445,000	4,555,000
Stanstead granite	5,685.000	0·2585	2,258,700	3,940,000
Nepheline syenite	9,13[illegible],500	0·2560	3,635,000	6,237,500
New Glasgow anorthosite	11,960,000	0·2620	4,750,000	8,368,000
Mount Johnson essexite	9,746,000	0·2583	3,872,600	6,750,000
New Glasgow gabbro *	15,650,000	0·2192	6,365,000	9,555,000
Sudbury diabase	13,763,000	0·2840	5,364,000	10,626,500
Ohio sandstone	2,290,000	0·2900	888,000	1,816,000
Plate glass	10,500,000	0·2273	4,290,000	6,448,000

Summary of Results (average) expressed in C.G.S. Units.

	E	σ	C	$D=\frac{1}{3}\left(\frac{m}{m-2}\right)E$
Wrought iron	$19{\cdot}37 \times 10^{11}$	0·2800	$7{\cdot}500 \times 10^{11}$	$14{\cdot}680 \times 10^{11}$
Cast iron	$10{\cdot}34 \times 10^{11}$	0·2500	$4{\cdot}132 \times 10^{11}$	$6{\cdot}897 \times 10^{11}$
Black Belgian marble	$7{\cdot}24 \times 10^{11}$	0·2780	$2{\cdot}982 \times 10^{11}$	$5{\cdot}736 \times 10^{11}$
Carrara marble	$5{\cdot}54 \times 10^{11}$	0·2744	$2{\cdot}171 \times 10^{11}$	$4{\cdot}090 \times 10^{11}$
Vermont marble	$5{\cdot}24 \times 10^{11}$	0·2630	$2{\cdot}069 \times 10^{11}$	$3{\cdot}680 \times 10^{11}$
Tennessee marble	$6{\cdot}21 \times 10^{11}$	0·2513	$2{\cdot}482 \times 10^{11}$	$4{\cdot}115 \times 10^{11}$
Montreal limestone	$6{\cdot}35 \times 10^{11}$	0·2522	$2{\cdot}504 \times 10^{11}$	$4{\cdot}250 \times 10^{11}$
Baveno granite	$4{\cdot}71 \times 10^{11}$	0·2528	$1{\cdot}875 \times 10^{11}$	$3{\cdot}178 \times 10^{11}$
Peterhead granite	$5{\cdot}71 \times 10^{11}$	0·2112	$2{\cdot}340 \times 10^{11}$	$3{\cdot}300 \times 10^{11}$
Lily Lake granite	$5{\cdot}63 \times 10^{11}$	0·1982	$2{\cdot}330 \times 10^{11}$	$3{\cdot}103 \times 10^{11}$
Westerly granite	$5{\cdot}09 \times 10^{11}$	0·2195	$2{\cdot}080 \times 10^{11}$	$3{\cdot}029 \times 10^{11}$
Quincy granite (1)	$4{\cdot}64 \times 10^{11}$	0·2152	$1{\cdot}916 \times 10^{11}$	$2{\cdot}750 \times 10^{11}$
Quincy granite (2)	$5{\cdot}68 \times 10^{11}$	0·1977	$2{\cdot}373 \times 10^{11}$	$3{\cdot}140 \times 10^{11}$
Stanstead granite	$3{\cdot}92 \times 10^{11}$	0·2585	$1{\cdot}556 \times 10^{11}$	$2{\cdot}718 \times 10^{11}$
Nepheline syenite	$6{\cdot}29 \times 10^{11}$	0·2560	$2{\cdot}505 \times 10^{11}$	$4{\cdot}290 \times 10^{11}$
New Glasgow anorthosite	$8{\cdot}25 \times 10^{11}$	0·2620	$3{\cdot}275 \times 10^{11}$	$5{\cdot}760 \times 10^{11}$
Mount Johnson essexite	$6{\cdot}71 \times 10^{11}$	0·2583	$2{\cdot}670 \times 10^{11}$	$4{\cdot}650 \times 10^{11}$
New Glasgow gabbro *	$10{\cdot}80 \times 10^{11}$	0·2192	$4{\cdot}380 \times 10^{11}$	$6{\cdot}589 \times 10^{11}$
Sudbury diabase	$9{\cdot}49 \times 10^{11}$	0·2840	$3{\cdot}700 \times 10^{11}$	$7{\cdot}329 \times 10^{11}$
Ohio sandstone	$1{\cdot}58 \times 10^{11}$	0·2900	${\cdot}612 \times 10^{11}$	$1{\cdot}250 \times 10^{11}$
Plate glass	$7{\cdot}24 \times 10^{11}$	0·2273	$2{\cdot}960 \times 10^{11}$	$4{\cdot}439 \times 10^{11}$

If the nepheline syenite be included with the basic rocks, an average value of D is obtained of 8,308,000.

This omits from consideration the sandstone, it being a rock

---

* See page 112.

of an entirely different class from the others, and furthermore one which shows so much hysteresis that the application of this method to it is less satisfactory than in the case of the other rocks of the series.

These results may be presented as follows:

	Average of D
Marbles and limestones	6,345,000
Granites	4,399,000
Basic intrusives	8,308,000

The cause of the much greater compressibility of granite as compared with the marbles and basic intrusives is not clear, but would seem to be connected with the presence of quartz. The only determination of the cubic compressibility of quartz, so far as can be ascertained, is one by Voigt,* the value obtained being 5,504,190 pounds (387×10^6 grams per. sq. cent.). This compressibility, as will be seen, is much greater than that found in the case of either the limestones or the basic intrusives, and while not in itself sufficiently great to account for the high compressibility of the granites, goes to show that in the quartz we have a mineral which is more compressible than the ordinary rock-making minerals which form the chief constituent in the rocks of the series examined.

The marbles and the limestones of the earth's crust are confined to its most superficial portion, resulting as they do from the process of sedimentation. There is every reason to believe, however, that what we may term the sub-structure of the earth's crust is composed of acid and basic plutonic igneous rocks. These make up the lowest part of the crust to which we have access, and are found coming up from the still greater depths.

The cubic compressibility, D, of the earth's crust must lie between the values given above for the granites and the basic intrusives, approaching one or other of these values according to the relative proportion in it of one or other of these classes of rocks.

If we take the average of the values obtained from these two classes of rocks as represented by the seven granites and the five basic intrusives (including the nepheline syenite), the value obtained for D is 6,353,500.

This, as will be seen, differs but little from the value of D obtained for plate glass, which is 6,448,000.

If, therefore, the earth's crust be composed of granite and basic igneous rocks in approximately equal proportions, its compressibility will be that of glass. If it be composed almost exclusively of granite, the earth's crust will be more compressible than glass; and if the basic rocks preponderate very largely, it will be less compressible than this substance.

---

* Quoted in Becker, Experiments on Schistosity and Slaty Cleavage, Bulletin 241, U. S. Geol. Survey, p. 32.

It is, however, in any case much more compressible than steel, which has a value for D of from 26,098,000 to 27,547,000 (18 to 19×10^{11}, C.G.S.)*.

The compression to which the rocks were subjected in this investigation ranged from 6,000 to 17,340 pounds to the square inch. Most of the rocks, however, were subjected to a load of from 9,000 to 15,000 pounds per square inch, and their bulk compression was determined for these loads as maxima. Higher pressures could not be employed without incurring the risk of breaking the specimen, and at the same time of destroying the measuring apparatus. One apparatus was in fact so destroyed.

The question arises as to whether under still higher pressures, if rupture could be avoided, the ratio of load to compression would be maintained. Judging from the deportment of much stronger substances, such as steel, when similarly tested, it is inferred that this ratio of bulk compression will remain constant for very much higher pressures, or until deformation sets in and the rock begins to flow.

With regard to the accuracy of the results obtained by this method as compared with those obtainable by any method in which cubic compression is actually produced and measured, it may be observed that by far the best method of this kind hitherto suggested seems to be that proposed by Richards and Stull.† We have endeavored to make use of this method in order to obtain results for purposes of comparison with those given in the present paper, but have not hitherto succeeded in overcoming certain experimental difficulties. The experimental errors in this method, though apparently small, still exist, and in applying it to rocks, which are much less compressible than the substances examined by Richards and Stull, these errors become proportionately more serious. Moreover, higher pressures than those used in the method employed in the present paper could scarcely be employed in this direct method, while difficulties dependent on the possible lack of absolute continuity in the substance of the rock and the danger of minute air-filled spaces, would probably present themselves in the case of most rocks. It seems that all things being considered, the indirect method here employed is probably as accurate as any direct method which can be used. The attempt to apply Richards and Stull's method to the same rocks is still being continued, however, and it is hoped that satisfactory results may be eventually obtained by its use.

McGill University, Montreal.

---

* Illustrations of the C.G.S. System of Units with tables of Physical Constants. Macmillan & Co., 1902, p. 60.

† New method of Determining Compressibility. Published by the Carnegie Institution, Washington, D. C., Dec., 1903 (No. 7).